

# Part I

## Finite-Dimensional Vector Spaces and the Fourier Transform

In this part we develop the mathematical framework of finite-dimensional Fourier transforms and give the basics of two fields where it has found fruitful application: in the analysis of coupled systems and in communication theory and technology.

Chapter 1 deals with complex vector analysis in  $N$  dimensions and leads rather quickly to the tools of Fourier analysis: unitary transformations and self-adjoint operators. The uncoupling of lattices representing one-dimensional crystals and electric  $RLC$  networks is undertaken in Chapter 2. We examine in detail the fundamental solutions, normal modes, and traveling waves for first-neighbor interactions in simple crystal lattices and extend these to farther-neighbor, molecular, and diatomic crystals. The Fourier formalism is also used to describe the analytical mechanics of these systems: phase space, energy, evolution operators, and other conservation laws. Chapter 3 introduces convolution and correlation, sketching their use in filtering, windowing, and modulation of signals and their detection in the presence of background noise. The workings of the fast Fourier transform (FFT) computation algorithm are given in Section 3.3. Finally, in Section 3.4, some properties of Fourier series and integral transforms (Parts II and III) are put in the form of corresponding properties of the finite Fourier transform on vector spaces whose dimension grows without bound.

Chapters 2 and 3 are independent of each other and can be chosen according to the reader's interest. With the first choice, Sections 1.6 and 1.7 will be particularly needed. The understanding of Chapter 3, on the other hand, does not require basically more than Sections 1.1–1.4. Before going

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to the following parts in this text, the reader may find Section 3.4 useful. Table 1.1, which gives the main properties of the finite Fourier transform, is placed at the end of Chapter 1.

